SIGNAL RECONSTRUCTION FROM SAMPLED DATA USING NEURAL NETWORK

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Abstract. For reconstructing the signal from sampling data, the method based on Shannon's Sampling theorem is usually employed. In this method, the reconstruction error appears when the signal does not satisfy the Nyquist condition. This paper proposes a new reconstruction method by using a linear perceptron and multi layer perceptron as FIR filter. The perceptron which has the weights obtained by learning in adapting the original signal suppresses the difference between the reconstructed signal and the original signal even when the Nyquist condition does not stand. Although the proposed method needs weight data, the total data size is much smaller than the ordinary sampling method, as the most suitable reconstruction filter is exclusively adapted to the given sampling data.

INTRODUCTION

Currently the method of reconstructing a signal from sampling signals is applied in various scenes. For reconstruction, a filter based on Shannon's Sampling theorem [6] is usually employed. The sampling theorem ensures complete reconstruction of a signal from sampled points whenever the Nyquist condition is satisfied. Violation of the Nyquist condition will cause aliasing phenomena, making it impossible to extract the frequency components that are needed to reconstruct the signal.

In the case of a musical Compact Disc(CD), the CD contains digital data obtained by sampling a particular musical signal. The sampling frequency is set to 44.1 kHz, considering that human audition is not sensitive to sound components that exceed 22 kHz.

Clearly there is a trade-off between the numbers of the sampling points, which is proportional to the sampling frequency and the high frequency components that can be reconstructed from those sample points, which influence the quality of the reconstructed signal.

In this paper, we propose a method of signal reconstruction using an adaptive signal reconstruction filter that is implemented through a neural network. The objective of our research is to reconstruct the signal even if it includes frequency components exceeding the Nyquist frequency.

There are some studies that have similar objectives with ours.

Candocia, et al.[2], proposed a method for obtaining a high-resolution image from its low-resolution counterpart, which is called "Super-resolution". In this method, the given low-resolution image is clustered into a number of classes, in which sub images with similar local information are grouped as a class. Each class will be linked to a pretrained neural network called linear associative memory(LAM) for reconstructing high resolution image based on the particular characteristic of the class. This method uses a group of neural networks that can be used for general images, while our method uses a neural network that is specialized to a specific data.

Ohira, et al.[5], proposed a reconstruction filter for high quality image enlargement, establishing a unique sampling function. Although both of this study and ours have similarity in generating a reconstruction filter, it differs from ours because it does not deal with missing sample points.

Katagishi, et al. [3] proposed a method for a unique sampling function for reconstruction filter. This study can be distinguished from ours, because in our research, we propose a method for generating an adaptive reconstruction filter that adaptively matches with the given sample points.

A method using Neural Network as FIR filter was proposed [4], in which the neural network predicts the length of the input points that should be used, which differs from ours in its objective.

In [1], a method of utilizing neural networks for images compression is proposed. The main difference of this research from ours is that, in our proposed research, the neural network is used for retrieving omitted sample point, so that the retrieved data satisfy the Nyquist condition for the original signal.

This paper is organized as follows. In Section 2 the formalization of the problem is explained. Section 3 explains the training of the neural network that can be utilized as a discrete time domain $f_\tau(t)$.

PROBLEM DEFINITION

The signal treated in this paper is illustrated in Figure 1. Suppose the original signal $f(t)$ satisfies the following condition:

$$F(\omega) \begin{array}{cl} \neq & 0 \ \ (|\omega| \leq W) \\ = & 0 \ \ otherwise \end{array}$$

where $F(\omega)$ is the Fourier Transformation of the original signal $f(t)$, and W is the Nyquist frequency of signal $f(t)$.

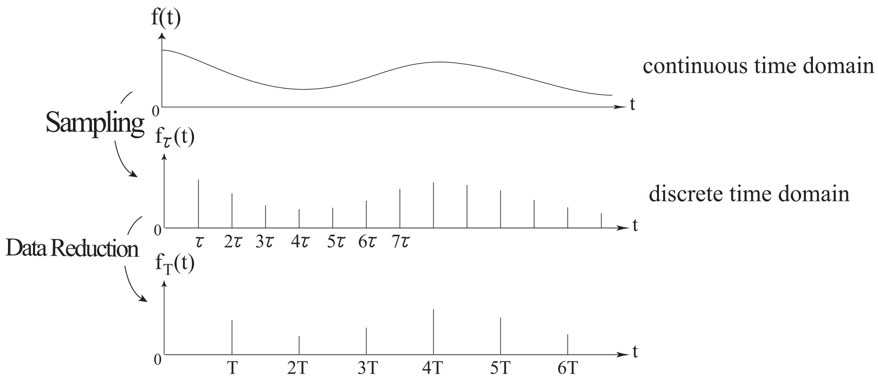

Figure 1: Sampling and Data Reduction

$f(t)$ is sampled with the sampling frequency of $\frac{1}{\tau}$ as,

$$\frac{2\pi}{\tau} \geq W, \tag{1}$$

so that we get sampled data $f_\tau(t)$ as

$$\begin{aligned} f_\tau(t) &= \sum_{n=0}^{\infty} \delta(t - n\tau) f(t), \\ \delta(t - n\tau) &= \begin{cases} 1 & (t = n\tau) \\ 0 & otherwise. \end{cases} \end{aligned} \tag{2}$$

For the purpose of data reduction, we define the sampled data $f_T(t)$ as,

$$f_T(t) = \begin{cases} f_\tau(t) & t = mT (m = 0, 1, 2, \cdots) \\ 0 & otherwise \end{cases}, \tag{3}$$

where

$$T = N\tau \ \ (N > 1, \ N : integer). \tag{4}$$

f_T will be stored as the sampled data, and used for the reconstruction of the original signal f_τ in the discrete time domain.

Our problem is to obtain a filter implemented with a neural network to reconstruct f_τ from f_T, in case that T and τ are defined as follows:

$$\frac{2\pi}{\tau} \geq W, \tag{5}$$

$$\frac{2\pi}{T} < W. \tag{6}$$

$f(t)$ can be always reconstructed from $f_\tau(t)$ because the condition (1) is satisfied. However $f_\tau(t)$ can not be reconstructed from f_T as $\frac{2\pi}{T}$ does not always satisfy the Nyquist condition. (If $\frac{2\pi}{T}$ satisfies the Nyquist condition, the FIR filter shown in Figure 2 can reconstruct f_τ completely.)

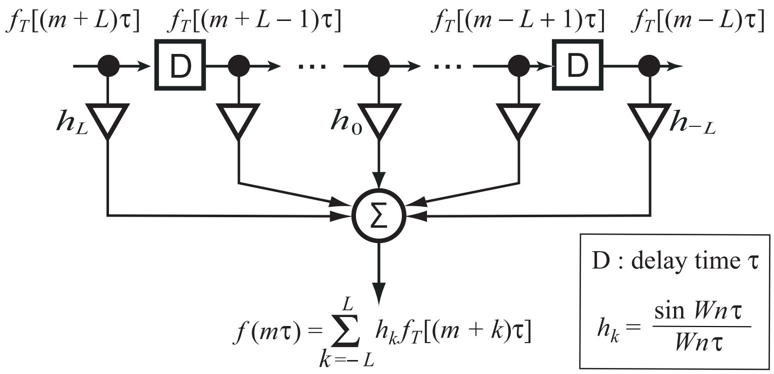

Figure 2: Reconstruction FIR Filter based on the Sampling theorem

NEURAL NETWORK AS RECONSTRUCTION FILTER

Transversal-type Perceptron is utilized as the filter to reconstruct f_τ from f_T shown in (3) and (4). We investigated the reconstructing method with linear perceptron and a Multilayer Perceptron (MLP) with non-linear hidden neurons.

Figure 3 shows the linear single layer perceptron. We expect that the data on the omitted sampling points can be constructed as follows:

$$O^{out}(m\tau) = \sum_{k=-L}^{L} W_k f_T[(m+k)\tau] \tag{7}$$

In the case of MLP with one middle layer, the reconstruction can be written as follows,

$$\begin{aligned} O^{out}(m\tau) &= \sum_{i=1}^{n_{mid}} W_i O^i(m\tau) \\ O^i(m\tau) &= g(\sum_{j=-L}^{L} V_{ji} f_T[(m+j)\tau]) \\ g(x) &= \frac{1}{1+exp(-x)} \end{aligned} \tag{8}$$

where W_k, V_{jk} are the connection weight between the k-th neuron in the middle layer and the output neuron, and the connection weight between the j-th input neuron and the k-th neuron in the middle layer, respectively. O^i is the output of the i-th middle neuron, while O^{out} is the output of the neural network.

The weights of the neural network should be corrected as to minimize the energy function E as follows,

$$E(t) = \frac{1}{2}(O^{out}(m+t) - f_\tau(m+t))^2, \tag{9}$$

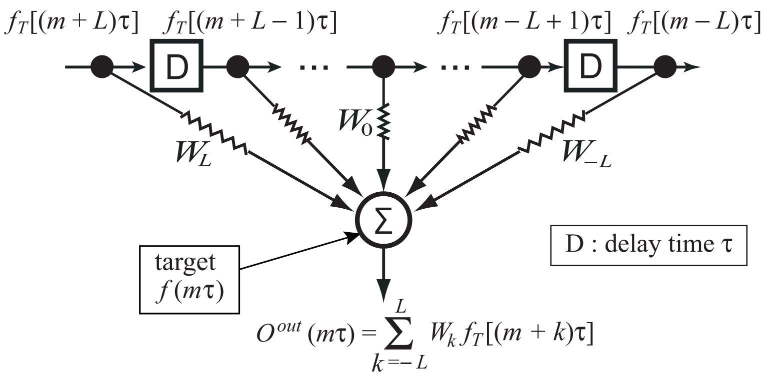

Figure 3: Linear Perceptron For Reconstruction

in which $E(t)$ is the error at the t-th training iteration. We utilize backpropagation training method. In the case of the linear perceptron,

$$W(t+1) = W(t) - \eta \frac{\partial E(t)}{\partial W(t)}, \tag{10}$$

and in the case of MLP,

$$W(t+1) = W(t) - \eta \frac{\partial E(t)}{\partial W(t)} \tag{11}$$

$$V(t+1) = V(t) - \eta \frac{\partial E(t)}{\partial V(t)}, \tag{12}$$

where $W(t)$ and $V(t)$ are the weight vectors of the perceptron after the t-th training.

Table 1: Parameter settings for Experiments

Parameters		Experiment 1		Experiment 2
original signal		0.1sec music signal		
T		$n\tau$ $(n = 2, 4, 6, 8)$		nτ $(n = 4, 6, 8)$
τ		1/44.1msec		
		input layer	hidden layer	input layer
number of units	linear perceptron	101	0	31 - 1501
	MLP	101	10	—

EXPERIMENTS

In these experiments a musical signal with a length of 0.1 sec was used. Utilizing the proposed neural network, we attempt to reconstruct the full

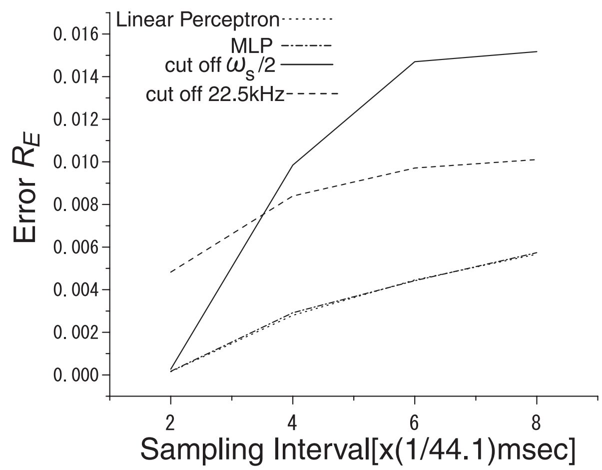

Figure 4: Error of reconstruction to some sampling interval

sample point data (the musical data sampled at 44.1 kHz), from the reduced sample point (with the sampling frequency of $\omega_s = \frac{44.1}{N} kHz$, $(N > 1)$).

The reconstructions from the reduced sample were done by using the linear perceptron and MLP. For comparison, we also reconstruct the data utilizing FIR with *sinc* function response as follows,

$$\begin{aligned} \hat{f}(\alpha\tau) &= \sum_{j=-L}^{L} f_T((\alpha - j)\tau) \frac{\sin(2\pi \frac{\Omega}{2} j\tau)}{2\pi \frac{\Omega}{2} j\tau} \\ \alpha &= 0, 1, 2, \cdots \end{aligned} \quad (13)$$

where $\hat{f}$ indicates the reconstructed signal, and L is set to 1000 in the experiment.

The error R_E between the original and reconstructed signal is calculated as follows,

$$R_E = \frac{\sum_{\alpha=1}^{N_{data}} (\hat{f}(\alpha\tau) - f_\tau(\alpha\tau))^2}{N_{data}} \quad (14)$$

In experiment 1, we compared the performance of the linear perceptron, MLP and the conventional FIR filter. Figure 4 shows the performance of respective reconstruction methods. "cut off $\omega_s/2$" and "cut off 22.5kHz" indicates the reconstruction error of $\hat{f}$ when $\Omega = \omega_s$, and $\Omega = 44.1kHz$, respectively where $\omega_s = 2\pi/T$. The result in Figure 4 shows that the neural network performed better.

In experiment 2, we reconstructed the signal with the proposed perceptron with regard to various numbers of input units when the sampling intervals were $\frac{4}{44.1}$ msec, $\frac{6}{44.1}$ msec and $\frac{8}{44.1}$ msec, respectively. Figure 5 shows the performance of the linear perceptron. The performance of MLP is almost equivalent to the linear perceptron as in experiment 1. The arrows in this

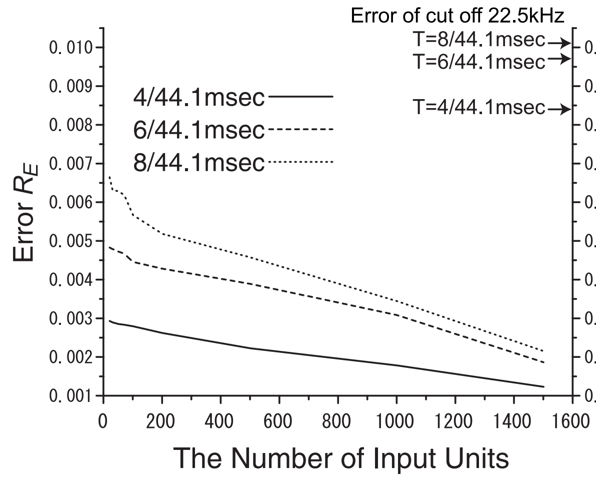

Figure 5: Reconstruction Error to the number of input units of linear perceptron

figure show the error of the FIR filter ($\Omega = 44.1kHz$) with the same input data. It is also shown that the performance of the proposed neural network is better. For example, the proposed neural network trained with the data sampled at $\frac{8}{44.1}$msec interval, performs better than the FIR filter with input data sampled at $\frac{4}{44.1}$ msec, implying that the neural network only requires half the data to reconstruct the signal compared with the FIR. The parameter settings for both of the experiments are shown in Table 1.

Performance of Linear Perceptron and MLP

We expected the MLP to perform better than the linear perceptron, because the MLP has the ability to emulate non-linear functions, while the linear perceptron can only express linear functions. But experimental results in Figure 4 shows that MLP is not superior compared with the linear perceptron, in case of musical signal used in these experiments.

The performance parity between the linear perceptron and MLP can be

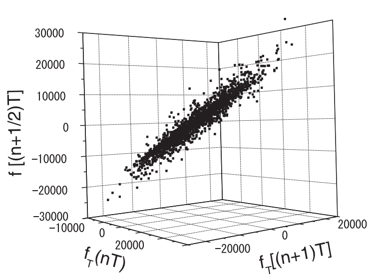

Figure 6: Distribution of $T = 2\tau$

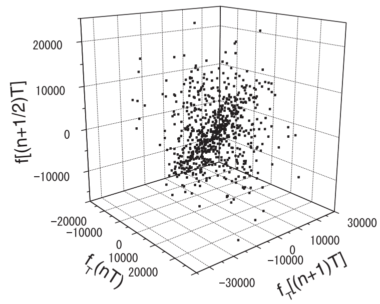

Figure 7: Distribution of $T = 6\tau$

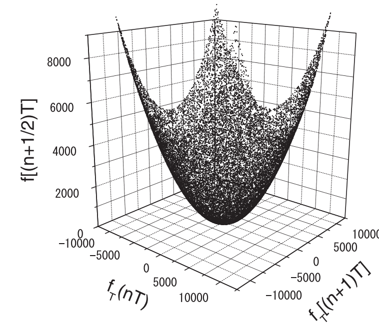

Figure 8: Distribution of non-linear data

explained by observing the distribution of sample data of musical signals. When the reconstruction with the perceptron with $\frac{T}{\tau}+1$ input units is attempted, it arises that the input signal and the target signal can be written for the intermediate omitted sampling point $(n+\frac{1}{2})T$ as follows:

$$\begin{cases} \text{input} & \big(f_T(nT), 0, 0, \cdots, 0, f_T[(n+1)T]\big) \\ \text{target} & f_\tau[(n+\frac{1}{2})T] \end{cases} \qquad (n = 0, 1, 2, \cdots). \qquad (15)$$

Then, if $\big(f_T(nT), f_T[(n+1)T], f[(n+\frac{1}{2})T]\big)$ are distributed around a linear plane with little insignificant variance, it is expected that the linear perceptron works well. Figures 6 and 7 shows the distribution of the sample points, when the sampling intervals are $T = 2\tau$ and $T = 6\tau$ (τ is the sampling interval that fulfills the Nyquist condition).

It is clear in Figure 6 that for $T = 2\tau$ that the samples are linearly distributed. Although as the sampling becomes sparse the variance becomes larger, the musical signal seems to be linearly predictable. This characteristics explains the performance parity between the linear perceptron and the MLP.

Although we assume that it is sufficient to utilize a linear perceptron to reconstruct such a signal, we may encounter data which are arbitrarily distributed. We tested the performance of MLP with artificially generated non-linear data as shown in Figure 8. The performance comparison between the MLP and the linear perceptron is shown in Table 2. It is clear that the MLP performs better compared with the linear perceptron with regard to non-linear data.

CONCLUSION

We have proposed a method of utilizing a neural network to deal with sparse data sampling that does not satisfy the Nyquist condition for signal reconstruction. The qualitative and quantitative performance evaluations show that the proposed method is better than the conventional FIR filter. For

Table 2: Result of Additional Experiment

	MLP	Linear Perceptron
error	2.0×10^{-5}	1.8×10^{-3}

example, a perceptron which has 31 input units can reconstruct the 44.1kHz-sampled data from the 5.52kHz-sampled data with less reconstruction error comparing the FIR filter reconstructing from 11.025kHz-sampled data as in Figure 5. As the length of the data is 0.1sec, the perceptron uses only 552+31 data units while the FIR filter uses 1102 data units for this reconstruction. We can draw the conclusion that with the proposed method, the size of the data can be compressed to 50% of the original data size. The propose method can be considered as a novel method for data reduction.

In the future, the proposed reconstruction method will be introduced to various applications, such as super-resolution imaging and super audio. Another possibility is to combine the proposed method with existing data compression methods such as MP3 and ATRAC.

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